PHYSICAL REVIEW B VOLUME 4, NUMBER 12 15 DECEMBER 1971

Dynamics of Localized Moments in Metals. III. Exchange Vertex Corrections and the Bloch Equation Parameters*

J. Zitkova, R. Orbach, and B. Giovannini†
Physics Department, University of California, Los Angeles, California 90024

(Received 29 July 1970)

The exchange vertex correction to the dynamic uniform transverse conduction-electron susceptibility $\chi_{ee}^{-+}(\omega)$ is calculated in a dilute magnetic alloy. The relation between the magnetic resonance linewidth $1/T_2$ and the longitudinal relaxation rate $1/T_1$, as calculated by Overhauser, is carefully examined. It is shown that $1/T_2$ equals the sum of the imaginary parts of the "up" and "down" conduction-electron self-energies plus the vertex correction. The latter equals in magnitude the frequency-modulation contribution to $1/T_2$ and its inclusion results in an equality of T_1 and T_2, guaranteeing rotational invariance. An additional feature of our results is that the form of $\chi_{ee}^{-+}(\omega)$ is that appropriate to exchange relaxation to the instantaneous local field.

I. INTRODUCTION

The conduction-electron self-energy in a dilute magnetic alloy has been computed by a number of authors.[1] In order to calculate the magnetic resonance properties, one must calculate the dynamic transverse susceptibility, using linear response theory, in terms of the two-particle propagator. Orbach and Spencer[2] (to be referred to as SO I and OS II) employed a decoupling procedure to compute the full coupled dynamic transverse susceptibility for such a system. However, there exists an unfortunate numerical error[3] in OS II. Upon its correction, one discovers that the OS II resonance linewidth is not equal to the sum of the imaginary parts of the one-electron self-energies. The decoupling procedure does allow for a distinction between the various contributions to the resonance linewidth. However, to display the origin of this difference, it is more transparent to use diagrammatic perturbation theory,[4] utilizing the one-electron propagators. We shall demonstrate that when vertex corrections to the susceptibility bubble are properly taken into account, an additional term enters the resonance denominator which adds to the one-electron self-energies and yields a resonance linewidth equal to the corrected OS II result. These corrections are evaluated by solving the integral Bethe-Salpeter equation for the vertex function. The form of the susceptibility is appropriate to relaxation towards the instantaneous local field. This result derives naturally from the finite-temperature diagrammatic methods[4,5] and is not "forced" as in the molecular field method.[6]

In Sec. II we shall carry through the perturbation theory for the conduction-electron transverse dynamic susceptibility to second order in the localized conduction-electron exchange coupling.

In Sec. III we shall analyze our results in terms of standard magnetic resonance notation, and display explicitly the character of the vertex correction. Our treatment is appropriate to high temperatures compared to the measuring or Zeeman frequencies.

We wish to emphasize that the purpose of this paper is to make clear the physical character of the many contributions to the conduction-electron resonance linewidth in a dilute magnetic alloy. We shall therefore take the localized spins to be in thermal equilibrium. In an actual magnetic resonance experiment, this is most definitely not the case. One must compute the full coupled susceptibility (OS II, for example) to obtain the full response function. Our analysis of the conduction-electron component alone is for the purpose of displaying the physics lying behind the linewidth appropriate to that component. We find it interesting that even in the lowest (second) order, where a contribution to the linewidth is obtained for an exchange coupling, two electron effects are essential to an understanding of the resonance properties. Without their inclusion, as we show in Sec. III, detailed balance is not satisfied, nor does the linewidth parameter $1/T_2$ agree with the spin-lattice relaxation rate $1/T_1$ as computed by Overhauser.[7]

II. CALCULATION OF DYNAMIC TRANSVERSE SUSCEPTIBILITY FOR CONDUCTION ELECTRONS

We shall compute in this section the transverse dynamic susceptibility for conduction electrons in a dilute alloy. Because the localized spins obey neither Bose nor fermion statistics, we must resort to a special method in order to treat their dynamics properly. We shall make use here of the "drone-fermion" representation developed for this problem by Spencer[8] though we should like to remark that we have obtained identical results using the method of Abrikosov[1] or of Brenig and Götze.[9]

Before we begin the actual computation, we should like to outline our procedure. We shall use standard finite-temperature perturbation diagrammatic methods for the calculation of the response function.[4,5] The vertex corrections are calculated using the usual Bethe-Salpeter self-consistency equation for the vertex function Λ. Evaluating Λ, we perform summations over the imaginary frequency variables in such a way that Λ will be given by an expression of exactly the same form as one-electron self-energies, in particular, their longitudinal part. Both the vertex corrections and self-energies are calculated to order J^2, where J is the (assumed) isotropic exchange integral between the localized and conduction-electron spins. Spatial averages are performed over an ensemble of random impurities. We adopt a constant density-of-states model for the conduction electrons for mathematical simplicity. We also restrict ourselves to the high-temperature regime, where kT is greater than either the measuring or Zeeman energies. This condition results in relaxation rates which are demonstrably not explicit functions of frequency. The form of our result is appropriate to relaxation to the instantaneous local field, a result which follows immediately from the perturbation diagrammatic method.[4,5]

To establish notation, we introduce the drone-fermion operators of Spencer for the localized spin $S_j = \frac{1}{2}$ at site j:

$$s_j^z = c_j^\dagger c_j - \tfrac{1}{2}, \quad s_j^+ = c_j^\dagger \phi_j, \quad \phi_j = d_j + d_j^\dagger, \tag{1}$$

where both the c and d operators are fermions. Their respective propagators will be labeled by C and D. The reader is referred to Spencer's paper for further background concerning the use of these operators. The total Hamiltonian is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1, \tag{2}$$

where $\mathcal{H}_0$ is the unperturbed Hamiltonian for the conduction and localized electrons in an external magnetic field h^z,

$$\mathcal{H}_0 = \sum_{\vec{p},\lambda} \xi_{\vec{p}\lambda} a_{\vec{p}\lambda}^\dagger a_{\vec{p}\lambda} + \omega_s S^z, \tag{3}$$

where $\lambda = \pm 1$, $\xi_{\vec{p}\lambda} = \xi_p - \mu + \frac{1}{2}\lambda\omega_e$, $\omega_e = g_e \mu_B h^z$, and $\omega_s = g_s \mu_B h^z$. The interaction Hamiltonian $\mathcal{H}_1$ is appropriate to an isotropic local exchange coupling and can be written as

$$\mathcal{H}_1 = -\frac{J}{N} \sum_{\vec{q},j} e^{-i\vec{q}\cdot\vec{R}_j} \vec{S}_j \cdot \vec{\sigma}_{\vec{q}} = -\frac{J}{N} \sum_{j,\vec{p},\vec{p}'} e^{-i\vec{R}_j\cdot(\vec{p}-\vec{p}')} \left(c_i^\dagger \phi_i a_{\vec{p}\downarrow}^\dagger a_{\vec{p}'\uparrow} + \phi_i c_i a_{\vec{p}\uparrow}^\dagger a_{\vec{p}'\downarrow} + \sum_\lambda \lambda (c_i^\dagger c_i - \tfrac{1}{2}) a_{\vec{p}\lambda}^\dagger a_{\vec{p}'\lambda} \right). \tag{4}$$

The zero-order propagators appropriate to the localized spins operators, and to the conduction electrons (denoted by $G_{\vec{p}\lambda}{}^{(0)}$), are

$$C^{(0)}(\bar{\nu}) = 1/(\omega_s - i\bar{\nu}),$$

$$D^{(0)}(\bar{\nu}) = 2/i\bar{\nu},$$

$$G_{\vec{p}\lambda}{}^{(0)}(\bar{\nu}) = 1/(\xi_{\vec{p}\lambda} - i\bar{\nu}), \tag{5}$$

where $\bar{\nu} = (2\nu + 1)\pi kT$, with ν taking all integer values. For completeness, we recompute below the first- and second-order self-energies for the conduction electrons appropriate to (4) computed first by Spencer[1]:

$$V_\lambda{}^{(1)}(\bar{\nu}) = c\lambda J \langle S^z \rangle, \tag{6a}$$

$$V_\lambda{}^{(2t)}(\bar{\nu}) = c\frac{J^2}{N} \sum_{\vec{p}'j} \sum_{\nu'\nu''} \frac{G_{\vec{p}'-\lambda}(\bar{\nu}') D(\bar{\nu}'') C(\bar{\nu} + \bar{\nu}'' - \bar{\nu}')}{(-i\beta)^2} = c\frac{J^2}{N} \sum_{\vec{p}'} \frac{f^+ f^+(\xi_{\vec{p}'-\lambda}) + f^- f(\xi_{\vec{p}'-\lambda})}{\xi_{\vec{p}'-\lambda} - i\bar{\nu} + \omega_s}, \tag{6b}$$

$$V_\lambda{}^{(2l)}(\bar{\nu}) = c\frac{J^2}{N} \sum_{\vec{p}'} \sum_{\nu'\nu''} \frac{G_{\vec{p}'-\lambda}(\bar{\nu}') C(\bar{\nu}'') C(\bar{\nu}'' - \bar{\nu}' + \bar{\nu})}{(-i\beta)^2} = \frac{cJ^2}{N} \sum_{\vec{p}'} \frac{f^- f^+}{\xi_{\vec{p}'\lambda} - i\bar{\nu}}. \tag{6c}$$

Here,

$$f^- = (1 + e^{\beta\omega_s})^{-1}, \quad f^+ = 1 - f^-, \quad \beta = (kT)^{-1},$$

and c is the fractional atomic concentration. The results have been obtained by converting the summations over $\bar{\nu}'$, $\bar{\nu}''$ to contour integrals. We have used the relations

$$N(x+y) = f(x)f(y)/[f^+(y) - f(x)],$$

where $N(x)$ and $f(x)$ are Bose and Fermi distribution functions, respectively, and

$$f'(x) = \lim_{\delta\to 0} \frac{f(x+\delta) - f(x)}{\delta} = -f^+(x)f(x).$$

The superscripts in (6) label both the order of the self-energy in powers of (4), and also the character of the process. Figure 1 displays the diagrammatic equivalents to (6). Thus, (6a) refers to the first order, or Knight shift, of the resonance. Expression (6b) refers to the mutual local-

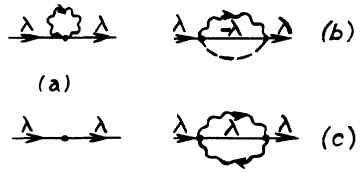

FIG. 1. Conduction-electron self-energies: (a) first-order contributions—$V^{(1)}$; (b) second-order contribution—$V^{(2t)}$; (c) second-order contribution—$V^{(2l)}$. The solid lines represent conduction-electron propagators, the wavy lines C, and the dashed D propagators.

ized conduction-electron (hence, transverse) spin-flip process, while (6c) refers to the z-z term of (4) (hence longitudinal). This distinction will be of some importance later in our analysis, for it will turn out that the lowest-order vertex correction will arise only from the latter process. The self-energies obey the relations

$$V_{-\lambda}(-x \pm i\delta) = -V_\lambda(x \mp i\delta) \ . \tag{7}$$

It is not difficult to carry out the integrations in (6) using a constant density-of-states model for the conduction electrons $[\rho(E) = \rho, \ -D \leq E \leq D]$.[1,2] We restrict our attention to high temperatures (ω_e, ω_s, $\omega \ll kT$) and note that for $S = \frac{1}{2}$ the fermion factors in (6) can be regrouped into localized spin functions. Then (6) reduces to

$$\begin{aligned}
&V_\lambda{}^{(1)} = c\lambda J \langle S^z\rangle \ , \\
&\mathrm{Re}\, V_\lambda{}^{(2t)} = 2c\lambda J^2 \rho \langle S^z\rangle \ln(\pi kT/2\gamma D), \\
&\mathrm{Re}\, V_\lambda{}^{(2l)} = 0 \ , \\
&\mathrm{Im}\, V_\lambda{}^{(2t)} = 2\pi c\rho J^2 f^- f^+ = 2\pi c\rho J^2[\langle (S^z)^2\rangle - \langle S^z\rangle^2] \ , \\
&\mathrm{Im}\, V_\lambda{}^{(2l)} = \pi c\rho J^2 f^- f^+ = \pi c\rho J^2[\langle (S^z)^2\rangle - \langle S^z\rangle^2] \ ,
\end{aligned} \tag{8}$$

where γ is Euler's constant $\cong 0.58$. The sum of all the terms in (8), which appear in the denominator of the second-order "dressed" Green's function $G_{\vec p\lambda}{}^{(2)}$, will be denoted by $V_\lambda{}^{(1+2)}(x \pm i\delta) = \lambda V \pm i\Gamma_e$. It is seen from (8) that V and Γ_e are functions of temperature only, and quite independent of frequency. This would not be true in the low-temperature limit, and would greatly complicate the analysis which follows.

The dynamic transverse susceptibility for the conduction electrons $\chi_{ee}{}^{-+}(\omega)$ is equal to the Fourier transform of the function

$$\chi_{ee}{}^{-+}(t) = i\theta(t)\langle [m_e^-(t), \ m_e^+(0)]\rangle.$$

It is obtained from analytic continuation above the real ω axis of the transform of the two-particle propagator

$$\mathfrak{F}_{ee}{}^{-+}(u) = \tfrac{1}{2}\langle T m_e^-(u) m_e^+(0)\rangle.$$

Thus,

$$\chi_{ee}{}^{-+}(\omega) = \mathfrak{F}_{ee}{}^{-+}(\omega + i0^+) \ .$$

The conduction-electron uniform magnetization is

$$m_e^\pm = g_e \mu_B \textstyle\sum_{\vec k} a_{\vec k\pm}{}^\dagger a_{\vec k\mp} \ .$$

The diagrammatic representation for the transform $\mathfrak{F}_{ee}{}^{-+}(\bar\nu_0)$ is displayed in Fig. 2, using dressed one-electron propagators and again the drone-fermion representation for the localized spins. The two-particle propagator, to order J^2, is given by

$$\mathfrak{F}_{ee}{}^{-+}(\bar\nu_0) = -\tfrac{1}{2}\mu_B{}^2 g_e^2 \frac{(-i)}{(-i\beta)} \sum_{\vec p, \nu} G_{\vec p\uparrow}{}^{(2)}(\bar\nu + \bar\nu_0) \times G_{\vec p\downarrow}{}^{(2)}(\bar\nu)\,\Lambda(\bar\nu, \ \bar\nu + \bar\nu_0) \ , \tag{9}$$

where $G_{\vec p\lambda}{}^{(2)}$ are the dressed propagators, with self-energies appropriate to (8), Λ is the vertex function, $\bar\nu = (2\nu + 1)\pi kT$, and $\bar\nu_0 = 2\nu_0\pi kT$, with ν and ν_0 taking all integer values. The integral equation for the vertex function will contain C-on propagator bubble $I(\bar\nu, \ \bar\nu + \bar\nu_0)$ pictured in Fig. 2(b). In precisely the same manner that V^{2l} was computed, we find

$$\begin{aligned}
I(\bar\nu, \ \bar\nu + \bar\nu_0) &= \textstyle\sum_j (1/\beta)\, e^{-i\vec R_j\cdot(\vec p - \vec p' - \vec p + \vec p'')} \\
&\quad \times \textstyle\sum_{\bar\nu''} C^{(0)}(\bar\nu'' + \bar\nu' - \bar\nu)\, C^{(0)}(\bar\nu'') \\
&= -cN\beta f^- f^+ \delta_{\bar\nu, \bar\nu'}\, \delta_{\vec p', \vec p''} \ .
\end{aligned} \tag{10}$$

Using this quantity, we can write an integral Bethe-Salpeter equation for the vertex function, depicted in Fig. 2(c):

$$\Lambda(\bar\nu, \ \bar\nu + \bar\nu_0) = 1 - \left(\frac{J}{N}\right)^2 \sum_{\vec p', \vec p'', \bar\nu'} \frac{i}{(-i\beta)} \times G_{\vec p'\uparrow}{}^{(2)}(\bar\nu' + \bar\nu_0)\, G_{\vec p''\downarrow}{}^{(2)}(\bar\nu')\, I(\bar\nu, \ \bar\nu + \bar\nu_0)\, \Lambda(\bar\nu', \ \bar\nu' + \bar\nu_0)$$

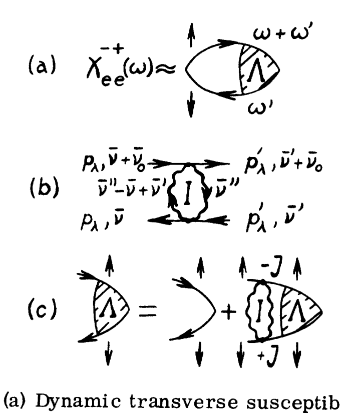

FIG. 2. (a) Dynamic transverse susceptibility $\chi_{ee}{}^{-+}(\omega)$. (b) The irreducible C-on propagators bubble $I(\nu, \nu + \nu_0)$. $\delta_{\nu, \nu'}\delta_{p', p''}$. The same I enters into a diagram representing $V^{(2l)}$ contribution to the self-energy. (c) Bethe-Salpeter equation for the vertex function Λ.

$$= 1 - \frac{cJ^2}{N} f^- f^+ \times G_{\vec{p}'\uparrow}{}^{(2)}(\bar{\nu} + \bar{\nu}_0)\, G_{\vec{p}'\downarrow}{}^{(2)}(\bar{\nu})\, \Lambda(\bar{\nu},\ \bar{\nu} + \bar{\nu}_0)\ . \tag{11}$$

Solving (11) for $\Lambda(\bar{\nu},\ \bar{\nu}+\bar{\nu}_0)$, and separating the product of the Green's functions by using partial fractions, one obtains

$$\Lambda(\bar{\nu},\ \bar{\nu}+\bar{\nu}_0) = \left[1 + \frac{(cJ^2/N)f^+f^-}{i\bar{\nu}_0 - \omega_e + V_\uparrow{}^{(1+2)}(\bar{\nu}+\bar{\nu}_0) - V_\downarrow{}^{(1+2)}(\bar{\nu})} \sum_{\vec{p}'} \left(\frac{1}{\xi_{\vec{p}'\lambda} - i(\bar{\nu}+\bar{\nu}_0) - V_\uparrow{}^{(1+2)}(\bar{\nu}+\bar{\nu}_0)} - \frac{1}{\xi_{\vec{p}'\lambda} - i\bar{\nu} - V_\downarrow{}^{(1+2)}(\bar{\nu})} \right)\right]^{-1} . \tag{12}$$

The product of $(cJ^2/N)f^-f^+$ with the two terms which are summed over $\vec{p}'$ can be identified immediately with $V_\lambda{}^{(2l)}$ as defined by (6c). This results in remarkable simplification, for we can write the vertex function entirely in terms of the self-energies:

$$\Lambda(\bar{\nu},\ \bar{\nu}+\bar{\nu}_0) = \left(1 + \frac{V_\uparrow{}^{(2l)}(\bar{\nu}+\bar{\nu}_0) - V_\downarrow{}^{(2l)}(\bar{\nu})}{i\bar{\nu}_0 - \omega_e + V_\uparrow{}^{(1+2)}(\bar{\nu}+\bar{\nu}_0) - V_\downarrow{}^{(1+2)}(\bar{\nu})}\right)^{-1} = \frac{i\bar{\nu}_0 - \omega_e + V_\uparrow{}^{(1+2)}(\bar{\nu}+\bar{\nu}_0) - V_\downarrow{}^{(1+2)}(\bar{\nu})}{i\bar{\nu}_0 - \omega_e + \tilde{V}_\uparrow(\bar{\nu}+\bar{\nu}_0) - \tilde{V}_\downarrow(\bar{\nu})}\ , \tag{13}$$

where we have written

$$\tilde{V}_\lambda(\bar{\nu}) = V_\lambda{}^{(1+2)}(\bar{\nu}) + V_\lambda{}^{(2l)}(\bar{\nu})\ .$$

Appearing in the denominator of the vertex function is the sum of the imaginary parts of the ↑ and ↓ conduction-electron self-energies [see Eq. (8)] together with the sum of the purely imaginary term $V_\lambda{}^{(2l)}(\bar{\nu})$ which arises from the integral equation determining the vertex function. This latter term happens to be exactly equal to the longitudinal part of the conduction-electron self-energy. We shall be able to identify in Sec. III each of these terms with appropriate parameters in the Bloch equations.

Now that we have determined the vertex function, we may return to (9) to obtain the dynamic conduction-electron transverse susceptibility from the relation $\chi_{ee}{}^{-+}(\omega) = \mathfrak{F}_{ee}{}^{-+}(\omega + i0^+)$. In (9), we convert the summation over $i\bar{\nu}$ to a contour integration in the complex ω' plane, the contour C going immediately below and above each of the cuts $\mathrm{Im}\,\omega' = 0$, $\mathrm{Im}(\omega' + i\bar{\nu}_0) = 0$. Changing variables when integrating along the latter cut from $\omega' \rightarrow \omega' - i\bar{\nu}_0$, ensuring analytic continuation of the integrand, and letting finally $i\bar{\nu}_0 \rightarrow \omega + i\delta$, we arrive at

$$\begin{aligned}\chi_{ee}{}^{-+}(\omega) &= \tfrac{1}{2}(\mu_B g_e)^2 (i/2\pi) \textstyle\sum_{\vec{p}} \int_C d\omega' f(\omega')\, G_{\vec{p}\uparrow}{}^{(2)}(\omega' + i\bar{\nu}_0)\, G_{\vec{p}\downarrow}{}^{(2)}(\omega')\, \Lambda(\omega',\ \omega' + i\bar{\nu}_0) \\ &= \tfrac{1}{2}(\mu_B g_e)^2 (i/2\pi) \textstyle\sum_{\vec{p}} \int_{-\infty}^{\infty} d\omega' \{\Lambda^{-+}(\omega',\ \omega'+\omega)\,[f(\omega') - f(\omega'+\omega)]\, G_{\vec{p}\uparrow}{}^{(2)}(\omega'+\omega+i\delta)\, G_{\vec{p}\downarrow}{}^{(2)}(\omega' - i\delta) \\ &\qquad - \Lambda^{++}(\omega',\ \omega'+\omega) f(\omega')\, G_{\vec{p}\uparrow}{}^{(2)}(\omega'+\omega+i\delta)\, G_{\vec{p}\downarrow}{}^{(2)}(\omega'+i\delta) \\ &\qquad + \Lambda^{--}(\omega',\ \omega'+\omega) f(\omega'+\omega)\, G_{\vec{p}\uparrow}{}^{(2)}(\omega'+\omega-i\delta)\, G_{\vec{p}\downarrow}{}^{(2)}(\omega'-i\delta)\}\ .\end{aligned} \tag{14}$$

Here, the $\Lambda^{\eta_1\eta_2}$ are the analytically continued functions $\Lambda(\bar{\nu},\ \bar{\nu}+\bar{\nu}_0)$ in (13):

$$\Lambda^{\eta_1\eta_2}(\omega',\ \omega'+\omega) = \Lambda(\omega' + i\eta_1\delta,\ \omega'+\omega+i\eta_2\delta)\ .$$

Their only discontinuities in the ω' plane lie along the lines $\mathrm{Im}\,\omega' = 0$, and $\mathrm{Im}(\omega' + i\bar{\nu}_0) = 0$. Separating the product of the Green's functions by again using partial fractions, and using analytically continued (13), we can write for (14) the expression

$$\begin{aligned}\chi_{ee}{}^{-+}(\omega) = -\tfrac{1}{2}(\mu_B g_e)^2 \frac{i}{2\pi} \sum_{\vec{p}} \int_{\infty}^{\infty} d\omega' \Bigg[&\frac{f(\omega') - f(\omega'+\omega)}{\omega_e - \omega - \tilde{V}_\uparrow(\omega'+\omega+i\delta) + \tilde{V}_\downarrow(\omega' - i\delta)} \left(\frac{1}{\omega' - \xi_{\vec{p}\downarrow} + V_\downarrow{}^{(1+2)}(\omega' - i\delta)} \right. \\ &\left. - \frac{1}{\omega'+\omega - \xi_{\vec{p}\uparrow} + V_\uparrow{}^{(1+2)}(\omega'+\omega+i\delta)} \right) - \frac{f(\omega')}{\omega_e - \omega - \tilde{V}_\uparrow(\omega'+\omega+i\delta) + \tilde{V}_\downarrow(\omega'+i\delta)} \\ &\times \left(\frac{1}{\omega' - \xi_{\vec{p}\downarrow} + V_\downarrow{}^{(1+2)}(\omega'+i\delta)} - \frac{1}{\omega'+\omega - \xi_{\vec{p}\uparrow} + V_\uparrow(\omega'+\omega+i\delta)} \right) + \frac{f(\omega'+\omega)}{\omega_e - \omega - \tilde{V}_\uparrow(\omega'+\omega-i\delta) + \tilde{V}_\downarrow(\omega - i\delta)}\end{aligned}$$

$$\times\left(\frac{1}{\omega' - \xi_{\vec{p}\downarrow} + V_{\downarrow}^{(1+2)}(\omega' - i\delta)} - \frac{1}{\omega' + \omega - \xi_{\vec{p}\uparrow} + V_{\uparrow}^{(1+2)}(\omega' + \omega - i\delta)}\right)\Bigg] \cdot \qquad (15)$$

Under the assumption that $\Gamma_e \ll kT$, we can make the approximation that

$$1/(x \pm i\Gamma) = P(1/x) \mp i\pi\delta(x)$$

in (15). In order to make our final result more transparent, we also rewrite $\tilde{V}_\lambda(x \pm i\delta)$ according to our earlier definition

$$\tilde{V}_\lambda(x \pm i\delta) = \lambda V \pm i(\Gamma_e + \Gamma_e^{(2l)}) \ . \qquad (16)$$

The last term in (16) is simply the longitudinal part of the conduction-electron self-energy and, as stated above, it arises from the vertex correction to the conduction-electron susceptibility bubble. The fact that $\Gamma_e^{(2l)}$ adds to Γ_e in (16) is of interest, for a cancellation would obtain for simple potential scattering. The opposite sign of the bubble vertices [see Fig. 2(b)] caused by the spin-dependent character of the interaction Hamiltonian is responsible for this important (indeed, crucial) difference.

Converting the sum over $\vec{p}$ to an integration over energy, we obtain the final expression for the dynamic transverse conduction-electron susceptibility to be discussed in detail in Sec. III:

$$\begin{aligned}\chi_{ee}^{-+}(\omega) &= (\tfrac{1}{2}\mu_B g_e)^2 \int_{-D}^{D} \rho\, d\xi_{\vec{p}} \left(\frac{-f(\xi_{\vec{p}\uparrow} - \omega - V) + f(\xi_{p\uparrow} - V) - f(\xi_{p\downarrow} + V) + f(\xi_{\vec{p}\downarrow} + \omega + V)}{\omega - \omega_e + 2V + 2i(\Gamma_e + \Gamma_e^{(2l)})}\right. \\ &\qquad \left. + \frac{f(\xi_{\vec{p}\uparrow} - \omega - V) - f(\xi_{\vec{p}\downarrow} + V) + f(\xi_{p\uparrow} - V) - f(\xi_{\vec{p}\downarrow} + \omega + V)}{\omega - \omega_e + 2V}\right) \\ &= -\tfrac{1}{2}(\mu_B g_e)^2 \rho \left(\frac{\omega}{\omega - \omega_e + 2V + 2i(\Gamma_e + \Gamma_e^{(2l)})} - 1\right) \\ &= \tfrac{1}{2}(\mu_B g_e)^2 \rho\, \frac{-\omega_e + 2V + 2i(\Gamma_e + \Gamma_e^{(2l)})}{\omega - \omega_e + 2V + 2i(\Gamma_e + \Gamma_e^{(2l)})} \ . \end{aligned} \qquad (17)$$

III. ANALYSIS OF $\chi_{ee}^{-+}(\omega)$ IN TERMS OF BLOCH EQUATION PARAMETERS

Overhauser[6] has computed the spin-lattice relaxation rate $1/T_1$ for conduction electrons scattering off of localized impurities in a metal via a scalar spin coupling. If we designate the *one*-electron contributions to the linewidth [the term $2\Gamma_e$ in (17)] by the symbol $1/T_2^*$, a comparison with Overhauser's results yields

$$\begin{aligned} 1/T_2^* &= \pi c \rho J^2 [\langle (S^z) \rangle - \langle S^z \rangle^2] \\ &= \tfrac{3}{4}(1/T_1) \ . \end{aligned} \qquad (18)$$

To carry our analysis further, we note that $1/T_2^*$ itself is composed of two parts: $2\,\mathrm{Im}\, V_\lambda^{(2t)}$ and $2\,\mathrm{Im}\, V_\lambda^{(2l)}$. We make the usual magnetic resonance identification: $1/T_1'$ is the spin-flip part of the linewidth; $1/T_2'$ is the longitudinal or frequency modulation part of the linewidth. Then we see immediately that $1/T_1' = 2\,\mathrm{Im}\, V_\lambda^{(2t)} = 1/2T_1$, but that if we identify $1/T_2'$ with $2\,\mathrm{Im}\, V_\lambda^{(2l)}$, we should obtain the incorrect result that $1/T_2' = 1/2T_1' = 1/4T_1$. Very general theorems require that, at high temperatures, isotropic systems, and in the short correlation time limit, all relevant to the system we are treating, $1/T_2'$ must equal $1/T_1' = 1/2T_1$, so that the sum $1/T_2' + 1/T_1' = 1/T_2$, the resonance linewidth, equals $1/T_1$. Returning to (17) we see that inclusion of the vertex correction, representing a two-electron contribution to the magnetic resonance linewidth, adds an imaginary term to the resonance denominator equal to the frequency-modula-self-energies, $2\,\mathrm{Im}\, V_\lambda^{(2l)}$. Noting that this term equals in magnitude $\frac{1}{2}(2\,\mathrm{Im}\, V_\lambda^{(2t)} = 1/2T_1)$ we have immediately the required result that the full linewidth in (17) is given by the expression

$$1/T_2 = 1/T_2^* + 1/4T_1 = 1/T_1 \ . \qquad (19)$$

Hence, the contribution of the vertex correction, acting through the frequency-modulation channel, has restored rotational invariance to the magnetic resonance system: The transverse relaxation time T_2 equals the longitudinal relaxation time T_1. We find it interesting that one must properly include two-electron effects to recover the result (19).

Another feature of (17) which is important is its form. The presence in the numerator of both the width and renormalized resonance frequency is indicative of a susceptibility where relaxation to the instantaneous local field has been required. Indeed, the results of OS I have exactly the structure of (17), and a detailed discussion of the consequences is contained in their paper. Remembering that (17) applies to exchange scattering, we see that it is permissible in an equation-of-motion approach to require exchange relaxation to the instantaneous local field, as was, in fact, done by the "Geneva group"[6] using molecular field theory, and by Langreth *et al.*[6] using Green's functions. We also see that their use of $1/T_1$ for the magnitude of the exchange relaxation rate for their line-shape calculation is correct, because, in fact, at high temperatures $1/T_2$ equals $1/T_1$ from (19). This is not a trivial point, for the physical justification of detailed balance is on the basis of "loss" of magnetization, i.e., on a longitudinal relaxation basis. However, the resonance equations display a resonance linewidth appropriate to a transverse relaxation rate $1/T_2$. The equality of these two rates at high temperatures allows for their procedure. Note, however, that at low temperatures, where the frequency dependence of the self-energies become important, one cannot even define a transverse relaxation rate $1/T_2$.[10]

IV. CONCLUSION

We have demonstrated that the two-particle character of the transverse dynamic conduction-electron susceptibility $\chi_{ee}^{-+}(\omega)$ necessitates inclusion of the vertex correction to establish rotational invariance [equivalence of the transverse ($1/T_2$) and longitudinal ($1/T_1$) relaxation rates]. The vertex correction itself is obtained by solving the Bethe-Salpeter integral equation. It arises from longitudinal fluctuations of the localized spin, and is equal in magnitude to the sum of the imaginary parts of the conduction-electron self-energies arising from those fluctuations. The perturbation methods result in a form for $\chi_{ee}^{-+}(\omega)$ which is characteristic of relaxation to the instantaneous local field, guaranteeing positive-definite resonance absorption[6] and correct zero frequency behavior.[2]

ACKNOWLEDGMENTS

We wish to thank Professor D. Scalapino for his remarks which initially guided us in this work, and Stewart E. Barnes who first brought to our attention the presence of the vertex contribution to the resonance linewidth in OS II. The perturbation method employed in the present paper is similar to a procedure developed by one of us (J. Z.), and S. E. Barnes (unpublished) applicable to the full coupled resonance properties of dilute magnetic alloys.

---

*Supported in part by the National Science Foundation Grant No. GP 21,290 and the U.S. Office of Naval Research Contract No. N00014-69-A-0200-4032.

†Present address: Ecole de Physique, Université de Genève, Genève 4, Switzerland.

[1] A. A. Abrikosov, Physics (N.Y.) 2, 5 (1965); H. J. Spencer, Phys. Rev. 171, 515 (1968); Yung-Li Wang and D. J. Scalapino, *ibid.* 175, 734 (1968).

[2] H. J. Spencer and R. Orbach, Phys. Rev. 179, 683 (1969), referred to as SO I in the text; R. Orbach and H. J. Spencer, *ibid.* 179, 690 (1969), referred to as OS II in the text.

[3] In OS II, an unfortunate numerical error has been discovered by us. In the process of decoupling, OS II on decoupling the exact equation of motion (2.20) in their paper to yield Eq. (2.22a) somehow obtained only one-half the correct contribution from the longitudinal localized spin fluctuations. We have recomputed their expression, and find that the last term in their (2.22a) and (2.22d) should be doubled. This has the effect of doubling the contribution from their $\Lambda_L(\omega)$ everywhere it appears. It also brings into agreement the OS II result for the conduction-electron resonance linewidth with that presented in this paper, and consequently with the spin-lattice relaxation rate $1/T_1$ so that detailed balance obtains as well.

[4] A. A. Abrikosov, L. P. Gorkov, and I. E. Dzialoshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, N. J., 1963).

[5] We have found of great help the paper of T. Holstein [Ann. Phys. (N.Y.) 29, 410 (1964)] employing the finite temperature diagrammatic methods for calculations of response functions. We should also like to take note of the recent paper by M. B. Walker [Phys. Rev. B 1, 3690 (1970)] which treats the localized spins in a dilute magnetic alloy.

[6] M. Peter, J. Dupraz, and H. Cottet, Helv. Phys. Acta 40, 301 (1967); P. Donzé and M. Peter, *ibid.* 40, 357 (1967); H. Cottet and M. Peter, *ibid.* 40, 361 (1967); B. Giovannini, Phys. Letters 26A, 80 (1967); H. Cottet, P. Donzé, J. Dupraz, B. Giovannini, and M. Peter, Z. Angew. Phys. 24, 249 (1968); J. Dupraz, thesis (University of Geneva, 1969) (unpublished). The first attempt to treat the problem from a microscopic point of view was that of B. Giovannini and S. Koide, Progr. Theoret. Phys. (Kyoto) 34, 705 (1965); and B. Giovannini, M. Peter, and S. Koide, Phys. Rev. 149, 251 (1966). See also the independent work of D. C. Langreth, D. L. Cowan, and J. W. Wilkins, Solid State Commun. 6, 131 (1968).

[7] A. W. Overhauser, Phys. Rev. 89, 689 (1953).

[8] Reference 1. In contrast to his preference of working in the time domain, we have chosen to use the drone-fermion approach in the usual imaginary frequency Fourier transform space (Ref. 4).

[9] W. Brenig and W. Götze, Z. Physik 217, 188 (1968).

[10] B. Giovannini and R. Orbach, Phys. Rev. B 2, 4517 (1970).